Issues in Indian Metrology, from Harappa to Bhāskarāchārya*

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ABSTRACT

Numerous systems of units were developed in India for lengths, angles, areas, volumes, time or weights. They exhibit common features and a continuity sometimes running from Harappa to Bhaskaracharya, but also an evolution in time and considerable regional variations. This paper presents an overview of some issues in Indian metrology, especially with regard to units of length and weight, some of which are traceable all the way to the Indus-Sarasvati civilization. It discusses, among others, the *aṅgula* and its multiple variations, and the value of *yojana* and its impact on calculations for the circumference of the Earth.

Keywords : Metrology, Linear unit, Harappan town-planning, Indus civilization, Siddhantic mathematics, Angula, Yojana.

From the late Vedic age to the pre-colonial era, numerous metrological systems were developed in India for units of lengths, angles, areas, volumes, time and weights. In this paper, I will focus on linear unit systems and discuss a few peculiar issues involving units from the *aṅgula,* the universal digit (with a wide range of definitions and values), to the *yojana*, a unit corresponding to the distance covered in a day by a pair of yoked bullocks (one of *yojana*'s meanings is 'yoked'). Some of those issues have remained unresolved; this paper hopes to shed some fresh light on them, in part by bringing into play data from archaeology.

Let us first bring together (Table 1) a selection of a few linear units defined in various texts. Although most of them are found scattered and usually undefined in the Vedic *saṁhitās* or their commentaries (notably the Brāhmaṇas), it is in the *Śulbasūtras* (texts dated between the eighth and the fifth century BCE which expound

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principles and rules of geometry for the construction of fire altars) that we find the first definition of a system of units. It is expanded and modified in later works, from Kauṭilya's *Arthaśāstra* to Bhāskarāchārya's *Siddhāntaśiromaṇi,* and in texts dealing with architecture, Ayurveda and other technical topics. More variants can be found in the Purāṇas, in Jain texts, and later in medieval works such as Abul Fazl's *Ain-i-Akbari*.

Unit/Author	**Baudhāyana's *Śulbasūktra*[1]**	**Kauṭilya's *Arthaśāstra*[2]**	**Āryabhaṭa's *Āryabhaṭīya*[3]**	**Varāhamihira's *Bṛhat Saṁhitā*[4]**	**Bhāskara's *Līlāvatī*[5]**
digit	1 *aṅgula* = 14 *aṇu* (millet grain)	1 *aṅgula* = 8 *yava* (barley grain)	*aṅgula* (undefined)	1 *aṅgula* = 8 *yava* (barley grain)	1 *aṅgula* = 8 *yava* (barley grain)
palm (4 digits)		1 *dhanurgraha*			
hand span (12 digits)	1 *prādeśa*	1 *vitasti*			
foot	1 *pada* = 10 *aṅgula* (small) or 15 *aṅgula* (big)	1 *pada* / *śama*/*śala* = 14 *aṅgula*			
cubit	1 *aratni* = 24 *aṅgula*	1 *aratni* / *hasta* = 24 *aṅgula*			1 *hasta* = 24 *aṅgula*
step	1 *prakrama* = 30 *aṅgula*				
man's height (short)				84 *aṅgula*	
man's height (medium)		1 *daṇḍa* / *dhanus*/ *nālikā* / *pauruṣa* (= 4 *hasta* = 96 *aṅgula)*	*1 nṛ* = 96 *aṅgula* = 4 *hasta*	96 *aṅgula*	1 *daṇḍa* = 4 *hasta* (= 96 *aṅgula*)
man's height (tall)		1 *dhanus* = 108 *aṅgula* (for roads /city walls); 1 *pauruṣa* = 108 *aṅgula* (for fire altars)		108 *aṅgula*	
man with arms stretched above	1 *puruṣa* = 120 *aṅgula*				
rajju		1 *rajju* = 10 *daṇḍa*			
goruta*/*krośa		1 *goruta* = 2,000 *dhanus* (of 4 *hasta*)			1 *krośa* = 2,000 *daṇḍa*
yojana		1 *yojana* = 4 *goruta* (= 8,000 *daṇḍa*)	1 *yojana* = 8,000 *nṛ*		1 *yojana* = 4 *krośa* (= 8,000 *daṇḍa*)

Table 1: A few linear units used in India (note that each of the above authors lists many more units than shown here).

While above table reflects a broad consensus, it also conceals widespread variations, some of which we will focus on here.[6] Before we do so, let us note in passing that the basic Indian definitions were shared by several other cultures, though with some differences. In the Egyptian system, a span was also 12 digits, but there were two cubits, a small one of 24 digits and a royal one of 28 digits, calculated to be 52.4 cm (leading to a digit of 18.7 mm). The Babylonian cubit of the same order at about 53 cm, but was however divided into 30 digits, implying a smaller digit (17.7 mm). The Greek digit of 19.3 mm leads to an Olympic cubit of 24 digits or 46.3 cm,[7] although other Greek cubits of 28 and 30 digits range from 52.7 to 53.3 cm.[8] The Latin system, as expounded by the celebrated architect Vitruvius,[9] was founded on a palm of 4 digits, a cubit of 6 palms (thus 24 digits), a foot of 4 palms (16 digits) related to man's height in the proportion of one to six (thus 96 digits for the height): this is identical to the dominant Indian system reflected in the above table.

Leaving those well-defined proportions aside, can we expect such precise values from the Indian system? Let us begin with the deceptively simple *aṅgula*.

The *aṅgula*

The *aṅgula* or digit is defined in the *Śulbasūtras*-s as the length covered by 14 grains of millet, while Kauṭilya and later authors prefer 8 grains of *yava* or barley (placed widthwise, not lengthwise). Attempts made in the 19th century to measure barley grains yielded a value of 0.77666 inch, or 19.7 mm.[10] But grains of cereals cannot be a very precise standard as they vary in size within a single crop, from one region or variety to another, and probably over time too. Perhaps for that reason, Kauṭilya (2.20.7) offers the alternative definition of 'the maximum width of the middle part of the middle finger of a middling man'.[11] This, however, is hardly an improvement: granting that the height of a 'middling' or average height can be estimated (we return to this issue in the next section), human hands are extremely variable irrespective of height. The matter is complicated[12] by parallel texts and traditions listing three *aṅgula*-s of 6, 7 or 8 grains of barley,[13] used for different purposes (e.g. public buildings and roads, private buildings, and furniture) or measured on the thumb of the *sthapati*,[14] the latter being one of the south Indian practices at least, where the *aṅgula*'s value could reach as much as 38.1 mm.[15]

If textual definitions do not yield a precise value, we must turn to empirical evidence. If we take a cubit, defined as the length from the elbow to the tip of the hand, to measure 18 to 22 inches and equate it with 24 *aṅgula*-s, we come to an *aṅgula* of 19 mm at the least, in consonance with the above-mentioned estimate for the barley measure. This may be why most scholars from J. F. Fleet in 1912 down took the *aṅgula* to be 'roughly equating ... ¾th of an inch,'[16] that is, 19.05 mm (which we will round off to 19 mm). Another reason might be that the traditional English 'finger' is the same three-fourths of an inch. The historians of science K. S. Shukla[17] and A. K. Bag,[18] as also the epigraphist Ajay Mitra Shastri,[19] to quote a few, endorsed this convenient value.

Some archaeological evidence can be marshalled in support of this value. The beautiful terracotta head of a three-eyed Shiva (Fig. 1a), found at Sringaverapura in Uttar Pradesh and datable to the first century CE, exhibits no fewer than 14 dimensions (out of 23 listed by B.B. Lal[20]) that are integral multiples (1, 2, 4, 5) of 19 mm, and

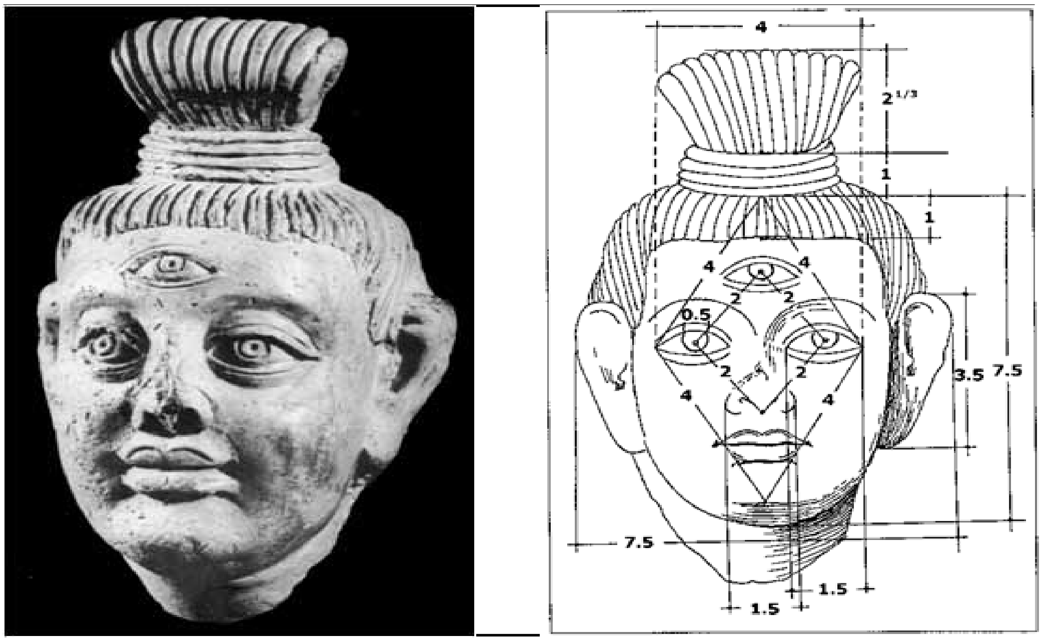

Fig. 1. (a) Terracotta head of Shiva, Sringaverapura; (b) Measurements in millimetres (Courtesy: Archaeological Survey of India); (c) Measurements in terms of 19 mm.

another 6 that are 1.5, 3.5 or 7.5 times 19 mm (Fig. 1b & 1c). Lal suggests a basic unit of 9.5 mm, which is of course possible (in that case the above 20 dimensions become integral multiples of it), although for historical reasons 19 mm seems likelier.

Another piece of evidence is supplied by the height and width of the characters of the Gupta–Brahmi inscription on the famous Delhi Iron Pillar, both of which have been measured by the metallurgist R. Balasubramaniam at 19.03 mm.[21]

Fig. 2. A close-up of the Gupta inscription on the Delhi Iron Pillar. (Courtesy: R. Balasubramaniam)

An alternate view was proposed by the metrologist V.B. Mainkar, who painstakingly traced the 'development of length and area measures in India' and narrowed the value of the *aṅgula* to 17.78 mm.[22] He also suggested a relationship with a graduated ivory rod found at the Harappan site of Lothal (Fig. 3), whose incised graduations occur every 1.77 mm. Since then, a rough graduated terracotta scale from Kalibangan (Fig. 4) has been analyzed by R. Balasubramaniam and J.P.

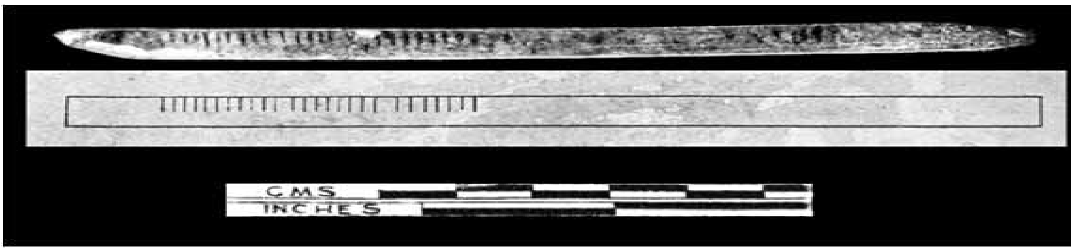

Fig. 3. The Lothal 'scale' (Courtesy: Archaeological Survey of India)

Fig. 4. The Kalibangan 'scale' (Courtesy: Archaeological Survey of India)

Joshi as being based on a unit of 17.5 mm.[23] My own work on Harappan linear units[24] has independently pointed to an *aṅgula* of 17.6 mm, which happens to be the average between the units obtained at Lothal and Kalibangan.

The Indologist Harry Falk, studying the Barabar caves of south Bihar, which date back to the 3rd century BCE, found their dimensions to be consistently integral multiples of a length ranging from 83.5 to 87 cm.[25] Falk comes to an average value of 85.5 cm, which he calls, with some justification, an 'Ashokan yard'.[26] Now, there exists, among the linear units not shown in Table 1, one of 48 *aṅgula*-s, that is, a half *dhanus* or *daṇḍa,* or again a double *hasta*; it is called *nādika* or *kiṣku* (with an alternate value of 42 *aṅgula*-s).[27] If we identify the Barabar caves' unit with it, this takes us to 17.81 mm for an *aṅgula*, close enough to the above values; opting for 1.9 cm would imply a unit of 45 *aṅgula*-s, which is unknown in the literature. Elsewhere, Falk proposes an *aṅgula* of 17.7 mm.[28]

So it would appear that we have, in the main, two different values in operation for the *aṅgula*, one of 19.0 mm and another of 17.7 mm, with the smaller value having roots in Harappan standards. From wholly different considerations rooted in the traditional values of various *hasta*-s, Raju and Mainkar reached a similar conclusion and proposed two *aṅgula*-s, one from Kauṭilya's time of 17.86 mm (which Mainkar later revised to 17.78 mm, as mentioned above) and another of later times equivalent to 20.6 mm (which, applying the same revision, should be 20.5 mm).[29] Their historical and region-wise analysis is probably the best we have today, although it still fails to account for an *aṅgula* of 19.0 mm (the difference of 7.5% with their 'upper' *aṅgula* is too high); moreover, the above archaeological evidence would tend to show that both values (17.7 and 19.0 mm) were in use right from Mauryan times on, the former possibly from Harappan times.

Let us further explore the issue of the *aṅgula*'s value by examining the *daṇḍa* or *dhanus*.

The *daṇḍa* or *dhanus*.

This is, let us recall, the height either or a bow or of a man. Interestingly, the Mahābhārata makes several references to bows measuring four cubits (i.e., 96 *aṅgula*-s), for 'normal' warriors, while the heights of Droṇa's bow as well as Arjuna's *gāṇḍīva* are six cubits (144 *aṅgula*-s), clearly to stress their eminence. A similar hierarchy exists in Indian iconography, which prescribes heights of 84, 96, 108 or 120 *aṅgula*-s for statues of deities, the middle two being the most common, and the last being reserved for major gods such as Rāma.[30]

Varāhamihira, who states in his *Bṛhat Saṁhitā* the above iconographic conventions, relates the first three heights with those of a short, medium or tall man, as shown in Table 1.[31] Let us examine this statement in the light of the two values we have proposed for the *aṅgula* (Table 2).

Height of a man	1 *aṅgula* = 17.7 mm	1 *aṅgula* = 19.0 mm
Short = 84 *aṅgula*-s	148.7 cm	159.6 cm
Medium = 96 *aṅgula*-s	169.9 cm	182.4 cm
Tall = 108 *aṅgula*-s	191.2 cm	205.2 cm

Table 2. Varāhamihira's three heights in *aṅgula*-s and their possible values in cm.

This table is revealing, for unless Varāhamihira had a purely notional approach in mind, we are forced to opt for the first set of values: 149, 170 and 191 cm are realistic values for a short, medium or tall man. The second set of values is unrealistically excessive. It may be objected that 84, 96, 108 *aṅgula*-s are indeed notional, since they are 7, 8 and 9 *vitasti*-s of 12 *aṅgula*-s, but this may just as well argue in support of practicality, especially in iconographic measurements: the three heights would easily be measured out with the sculptor's hand span.

We could in fact reverse the question and ask what an average human height in Varāhamihira's time was, then work out its 1/96th part to get to the *aṅgula*. I do not have direct data to offer, but a study of 260 skeletons from the Harappan civilization established that Harappan males 'had an average stature of 1691.87 mm', that is, 169.2 cm.[32] This yields an *aṅgula* of 17.63 mm, quite in tune with the first choice in Table 2 (central column).

B.B. Lal, in search for a unit system to explain the iconography of the Shiva head at Sringaverapura, recorded measurements of a dozen local young male inhabitants (Table 3).[33] The average height, 166.3 cm, is smaller than that of the Harappan skeletons, and takes us to an *aṅgula* of 17.3 mm. In any case, this argues against considering an *aṅgula* of 19 mm for Varāhamihira's three heights (Table 2).

B.B. Lal's table is important for another reason: if we consider other dimensions — the palm of 4, the span (*vitasti*) of 12 or the cubit (*hasta*) of 24 *aṅgula*-s — we obtain distinct values for the *aṅgula*, as the bottom row shows. This simply means that these bodily proportions are idealized: in practice, the cubit, for instance, is longer than two spans in most individuals: here the ratio is 45.24 to 21.3, i.e. 2.12, instead of a theoretical 2. This is easy to verify (in my own case, I found the ratio to be 2.23) and may explain the different values for the *aṅgula*: 17.7 mm is more compatible with the

Name	Locality	Age	Height	*4 aṅgulas*	*1 vitasti*	*1 hasta*
			96A	**4A**	**12A**	**24A**
1. Shri Shrinarayan	Sringaverapura	22	107.5	7.9	22.1	46.8
2. Shri Nand Lal	Ram Chaura	24	164.8	7.8	21.38	47.8
3. Shri Ram Raj Yadav	Moharabe	22	164.5	7.2	21.4	46.0
4. Shri Rama Shankar	Shyampur	19	168.5	7.3	22.2	46.0
5. Shri Lallan Pande	Sringaverapura	26	170	7.4	20.5	44.5
6. Shri Ramji Misra	Ram Chaura	28	168	7.3	21.0	46.4
7. Shri Phool Chand Misra	Milan-ka-pura	22	160	7.3	21.4	43.8
8. Shri Ram Singh Yadav	Kanjia	22	164	6.9	21.6	43.6
9. Shri Sukh Ram	Shyampur	25	165	7.9	20.0	42.3
10. Shri Onkar Nath Yadav	Guru-ka-pura	27	172	7.5	21.0	45.5
11. Shri Narendra Deo Misra	Bishanpura	24	159	6.5	18.0	42.0
12. Shri Ramesh Chandra Shukla	Sambharpur	24	169.0	7.4	24.3	48.2
Age in years; and height, etc. in centimetres	**Averages** :		**166.3**	**7.37**	**21.3**	**45.24**
Resulting *aṅgula* :			**1.73**	**1.84**	**1.77**	**1.88**

Table 3. Physical dimensions of Sringaverapura residents. (The letter A below the units in the first row stands for '*aṅgula*'.) The values in the last two rows are my addition.

theoretical definition of the body's height (*daṇḍa* or *dhanus*), while 19.0 mm rather fits that of the cubit (*hasta*). It also explains the variations in the brick sizes found at Sringaverapura[34] and other excavated cities, as the bricks would have been measured by hand (Lal makes a convincing case for the dimensions to be combinations of hand spans and palms).

Returning to the *daṇḍa* or *dhanus,* it is invariably approximated in recent literature to 6 feet. This is merely derived from the value of ¾ inch for the *aṅgula*: 0.75" x 96 = 72", which is 6 ft or 182.9 cm. This value, as we just saw, is not a realistic one for an average human height. Is there a way to assess the *daṇḍa* independently of skeletal data or of the *aṅgula*? I offer here a few approaches.

The first is my own study of the town planning at Dholavira, a major Harappan city in the Rann of Kachchh; almost all dimensions of fortifications, reservoirs and other structures turned out to be integral multiples of an unit of 190.1 or 190.4 cm, which I equated to 108 times 17.60 or 17.63 mm (my above-mentioned value for the Harappan *aṅgula*).[35] This leads us to a human height of 169 cm, perfectly in tune with the above-mentioned skeletal findings.

Recent work by Mohan Pant and Shuji Funo[36] compared the grid dimensions of building clusters and quarter blocks of three cities: Mohenjo-daro, Sirkap (Taxila, early historical), and Thimi (in Kathmandu Valley, a town of historical origins). After grids were superimposed on published plans of the three cities, block dimensions were found to measure 9.6 m, 19.2 m (= 9.6 m x 2), or multiples of such dimensions. This led them to a *daṇḍa* of 192 cm (which they equated to 108 *aṅgula*-s as prescribed in the *Arthaśāstra*). Pant's and Funo's value differs from mine by less than 1%; it is equivalent to a human height of 170.7 cm.

I mentioned earlier Harry Falk's 'Ashokan yard' of 85.5 cm, which has been equated to 48 *aṅgula*-s. The corresponding human height will be double that, i.e. 170 cm.

Lastly, R. Balasubramaniam studied the Delhi Iron Pillar (Qutub Minar complex) by applying to it the Harappan *dhanus* and *aṅgula* I had proposed in the Harappan context.[37] He found that the pillar's dimensions took simple expressions in terms in those units: for instance, its total length of 7.67 m is precisely 4 times 192 cm. Balasubramaniam extended his studies to caves of the Mauryan age and even the Taj Mahal complex, with promising results.[38]

These approaches converge towards a value of the *daṇḍa* (96 *aṅgula*-s, height of an average man) of 169 to 171 cm, while the extended *daṇḍa* (108 *aṅgula*-s) or height of a tall man works out to 190 to 192 cm.

The *yojana*

Although, again, the *yojana* was given several definitions, Table 1 reflects an apparent consensus from Kauṭilya to Bhāskarāchārya. Other Siddhāntic authors may be quoted in support; Mahāvīra (9th century), for instance, also takes the *yojana* to be 4 *krośa*-s of 2,000 *daṇḍa*-s or 8,000 *daṇḍa*-s (of 4 *hasta*-s).[39] The same tradition perdures in the Kerala School, all the way to Śaṅkaravarman's *Sadratnamālā* of the 19th century.[40] If we accept the above value for the *daṇḍa* (170 cm), the *yojana*'s value will be 13.6 km.

However, values covering a bewilderingly wide range have been offered in the course of time, from 4 to 12 miles (7 to 20 km) or more.[41] Some of them depend on alternative definitions for the *yojana,* others on the value of the *li,* a unit used by Chinese travellers to India to express distances between the cities they visited and other geographical features. Unfortunately, estimates for the *li* vary widely, as does its precise relationship to the *yojana,* which is variously stated to be 16, 30 or 40 *li*-s.

For instance, in the 7th century, Xuan Zang (Hsüan Tsang) explains, 'In the subdivision of distances, a *yôjana* is equal to eight *krôśas* (*keu-lu-she*); a *krôśa* is the distance that the lowing of a cow can be heard; a *krôśa* is divided into 500 bows (*dhanus*); a bow is divided into four cubits (*hastas*); a cubit is divided into 24 fingers (*aṅgulis*); a finger is divided into seven barleycorns (*yavas*).'[42] This is all familiar, except that it yields 4,000 *dhanus* for the *yojana* rather than the standard 8,000 *dhanus* of Siddhāntic literature. Other texts support this half-value (of about 6.8 km if we accept a *dhanus*/*daṇḍa* of 170 cm), such as the Buddhist text *Lalitavistara*,[43] which probably explains the choice of the Chinese travellers.

Writing in the 11th century CE, the Persian savant al-Bīrūnī also records in his travels through India the classical system for the lower units (from a grain of barley to a *dhanus* of four times 24 *aṅgula*-s), but notes a *krośa* of 25 times 40 = 1,000 *dhanus* and a *yojana* of twice that, i.e. 2,000 *dhanus* or one-fourth of the classical value or half of that in *Lalitavistara*.[44] Clearly, several different systems were in actual use.

The Earth's circumference

The *yojana* gets embroiled in the issue of the Earth's circumference. Measuring our planet has been a concern with many cultures of antiquity. As far as we know, Eratosthenes, chief librarian at the Library at Alexandria in the third century BCE, was the first to propose a method and a result. His method consisted in measuring the curvature of the globe by noting the angle (about 7° from the vertical) of the sun's rays at noon on the summer solstice at Alexandria, while at Syene in southern Egypt (modern Aswan), the same rays were perfectly vertical: it is easy to show that the ratio of the observed angle to 360° is the same as the ratio of the arc of circle between Syene and Alexandria (approximated to their known linear distance) to the Earth's circumference. However, owing to the uncertainty in the value of the stade or stadium, the linear unit used by Eratosthenes, it is impossible to derive an exact result, estimated by *Encyclopaedia Britannica* to be about 15% too large (if 1 stadium = 185 m, a generally accepted value).[45] In the second century BCE, Poseidonios measured the difference in the star Canopus's altitude measured from two different locations to propose another, less precise value.[46]

In India, about 500 CE, the first savant to propose a value was Āryabhaṭa in his *Āryabhaṭīya*, which states (1.7) that the Earth's diameter is 1,050 *yojana*-s.[47] (I will leave aside for now all discussion on the diameters of the Moon and Sun, which usually are evaluated in such discussions along with the Earth's.) Many recent scholars take off from the Earth's diameter or circumference as they have now been measured, and assuming Āryabhaṭa knew those true values, back-calculate the value of 'his' *yojana.* This is clearly not acceptable, as the assumption is illegitimate. We must proceed the other way, starting from the data Āryabhaṭa himself provides.

Āryabhaṭīya (2.10) states the value of π to be 3.1416, and as we saw adopts a *yojana* of 8,000 *nṛ*-s or human heights (of 1.7 m); this takes us to a circumference of 3298.68 *yojana*-s, or 44,862 km. The correct value is 40,075 km (at the equator), which means that *Āryabhaṭa's* estimate was about 11% too large, a meritorious result in the late 5th century CE, although not much of an improvement on Eratosthenes. However, *Āryabhaṭa's* extremely concise style does not include an explanation of how he arrived at that result.

In his *Pañcasiddhāntikā* (13.16 & 18), Varāhamihira, Āryabhaṭa's contemporary, states the Earth's circumference to be 8 8/9 *yojana*-s per degree of latitude, or 3,200 *yojana*-s in all,[48] that is, 43,520 km (off by 8.5%), thus a slight improvement. In his *Mahābhāskarīya* of the early 7th century, Bhāskara I, Āryabhaṭa's earliest commentator, returns (5.4) to his master's value of 1,050 *yojana*-s for the diameter.[49] Later in the same work, however, he offers (7.23) the value of 1,600 *yojana*-s, or 1.52 times the earlier value.[50] (Note that the diameters of the Sun and the Moon, which are listed together with that of the Earth, undergo a similar inflation.)

Bhāskara I's contemporary, Brahmaṇupta, writing his *Brāhmasphuṭasiddhānta* in 628 CE, attributes (1.36, 21.32) to the Earth a slightly smaller diameter of 1,581 and a circumference of 5,000 *yojana*-s.[51] (The value of π used here, 5000/1581 or 3.1625, is nothing but an approximation for $\sqrt{10}$, one of π's traditional values from early Jain texts onward.) The Earth's diameter is now 1581/1050 or 1.506 (roughly 3/2) times that of Āryabhaṭa's in terms of *yojana*-s.

The 9th-to-10th-century *Sūryasiddhānta* (1.59) draws from Bhāskara I's second value for the Earth's diameter, which it sets at 1,600 *yojana*-s, with the circumference at $\sqrt{10}$ times that.[52] Adopting that value of π yields a circumference of 5,059.6 *yojana*-s. (These figures were recorded by Samuel Davis as early as in 1789.[53]) Bhāskarāchārya in his 12th-century *Siddhāntaśiromaṇi* gives instead 1,581 and 4,967 *yojana*-s,[54] the first being Brahmaṇupta's value for the diameter (but here with 3.1416 for π, which corrects Brahmagupta's value for the circumference).

The Kerala School of Astronomy has sometimes been regarded as an improved extension of Āryabhaṭa's School, so it is no surprise to see its astronomers return to Āryabhaṭa's values for the Earth's dimensions. Nīlakaṇṭha Somayājī's *Tantrasaṅgraha* of 1500 CE states (1.29) that its circumference is 3,300 *yojana*-s.[55]

Let us summarize the above data in Table 4.

	Earth's circumference (in *yojana*)	**Earth's diameter (in *yojana*)**
Āryabhaṭa	1,050	[3,298.7]
Varāhāmihira	[1,011.9]	3,200
Bhāskara I	1,050 also 1,600	[3,298.7] also [5,026.6]
Brahmagupta	1,581	5,000
Sūryasiddhānta	1,600	[5,059.6]
Bhāskara II	1,581	4,967
Nīlakaṇṭha	[1,050.4]	3,300

Table 4. Dimensions of the Earth according to several savants and texts. The figures in square brackets are computed, not stated (with whatever value of π the author is known to be using), and rounded off to the first decimal.

DISCUSSION

Many scholars have posited a smaller *yojana* to try and account for the above two groups of values (if we broadly regard 1,581 and 1,600 as belonging to the same group). In effect measuring now 9.0 km instead of Āryabhaṭa's 13.6 km, it would need to be defined as (8,000 x 1,050)/1,581 = 5,313 *daṇḍa*-s. I am not aware of any such definition in the literature; *Lalitavistara* does define a *yojana* as 4,000 *daṇḍa*-s,[56] but that will not help us. Moreover, Table 1 reminds us that Bhāskarāchārya adopted the classical definition of 8,000 *daṇḍa*-s for the *yojana*: it looks as if he and Brahmaṇupta are silently using two different concepts for this unit.

The 19th-century historian of Indian astronomy S.B. Dikshit, for instance, acknowledged this duality and tried to define the *yojana* as 30 *li*-s in Hsüan Tsang's time (which is also Brahmagupta's or Bhāskara I's), with 6 *li*-s equal to 1 mile, so that a *yojana* becomes 5 miles or 8.05 km.[57] Dikshit points out that this value, combined with Brahmagupta's figure of 1,581 *yojanas*, 'is very nearly equal to that of the accurately established measure of the [Earth's] diameter', which is indeed correct within 0.3%. But his discussion of the value of 5 miles, based on scholarly estimates of the *li*, is speculative and appears influenced by the result obtained. No definition of the *yojana* in the Siddhāntic literature that I know of comes close to 5 miles, unless we are prepared to manipulate its subunits.

This, in a way, is what R. Balasubramaniam attempted.[58] In sum, he argued that two different values were used for the *hasta* (normally 24 *aṅgula*-s): 'The astronomers who estimated the earth's circumference as approximately 3300 *yojayams* have utilized the measure based on the 42-angulam *hasta* ..., while those who estimated it to be 5000 *yojanams* have used the measure based on the 28-angulam *hasta* ... The *angulam* remained the same at 1.763 cm.' The ratio of 42 to 28 is 1.5, which would indeed explain the ratio between the two groups in Table 4. However, the literature is silent on such choices, and a simple calculation shows that Āryabhaṭa's value for the Earth's circumference would now be much less precise at over 50,200 km. Nevertheless, Balasubramaniam's careful study deserves fresh attention and discussion, in my opinion.

In an unpublished paper,[59] K. Chandra Hari uses an astronomical argument that distinguishes between two schools of Indian astronomy, the *audayikā* (with the dawn reckoned from the mean sunrise at the equator) and the *ārdharātrikā* (with the day reckoned from the mean midnight at the equator). The parameters involved in astronomical computations are different in the two systems. According to Chandra Hari, 'It is ... evident that the yojana unit we find in Āryabhaṭīya is 1.5 times the yojana of Ārdharātrikā system. ... Use of two differing values as 1050 and 1600 for earth's diameter by an astute astronomer like Āryabhaṭa alone is sufficient to draw the inference that the absolute magnitude of yojana as a metrological unit had no relevance in astronomy.'

While Chandra Hari may come close to the reason for the dual system we have seen, and while it is true that most algorithms involved in calculations of eclipses only ask for relative values between the Earth's and the Moon's diameters, the absolute value of the Earth's diameter does intervene at some stages. Nevertheless, my cursory study of the astronomical literature also led me to suspect that the dual system has its roots in computational methods of Indian astronomy, especially as regards eclipses. It is best at this stage to leave the question open and invite experts to examine the above two papers and revisit the vexed question of India's *yojana*.

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